The World According to GARP*: Non-parametric Tests of Demand Theory and Rational Behavior

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Non-parametric Tests of Demand Theory and Rational Behavior

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Abstract.

The purpose of this paper is twofold. We first point out that violation of rationality axioms (SARP, GARP, WARP) do not necessarily lead to a non-rational behavior. Second, our tests of axioms SARP, GARP and WARP over a Polish panel data (1987-90) show that over the 3630 households only 240 violate the three axioms. However these 240 violations are not caused by the non-respect of demand theory axioms but by the changing of preferences over the period. A logistic regression confirms the robustness of the test since the more the real expenditure increases in absolute value, the more the probability of violating the axioms increases (the respect of the axioms by the 3390 households is not due to an increase of the real expenditure). Moreover, changing in the composition of the family structure increases the probability of being inconsistent. It seems therefore that the 240 apparent violations are due to the appearance of new constraints, which increase the shadow prices of the goods. In order to explain these 240 households' preference changes, we build an econometric model of prices including an observed monetary component and an unobserved non-monetary component expressing the constraints faced by the agent. The estimation of this econometric model shows that the agents who apparently violate the axioms have these complete price changes superior to those of the agents who respect the axioms. Thus the agents who apparently violate the axioms faced during the period a change of their non-monetary resources and the appearance of new constraints.

Résumé

Dans cet article nous montrons d'abord que la violation des axiomes de rationalité des préférences revelées (SARP, GARP, WARP) ne correspond pas nécessairement à des comportements non rationnels. Nous testons ces axiomes sur les données individuelles de 3630 ménages du panel polonais (1987-1990) ; 240 ménages violent les trois axiomes, mais on montre que ces violations ne sont pas la conséquence d'une contradiction aux axiomes de la théorie de la demande mais sont entraînés par la modification des préférences individuelles pendant cette période. Une analyse logistique montre que la probabilité de violation des axiomes augmente avec l'accroissement du revenu réel de ménage ce qui confirme la robustesse de ce test non-paramétrique de la rationalité. La probabilité de violation dépend également des modifications démographiques des ménages. On fait l'hypothèse que ces violations apparentes sont dues à l'apparition des nouvelles contraintes de choix qui se traduisent par l'apparition des prix virtuels. L'estimation de ces prix virtuels se fait par la considération des biais d'endogénéité dans l'estimation en cross-section, c'est à dire par la comparaison des estimations en coupe et en dynamique. L'étude confirme que les ménages qui violent apparemment ces axiomes de rationalité subissent des variations de prix virtuels nettement supérieures aux non-violeurs. Ceci semble indiquer qu'ils subissent pendant cette période des modifications plus fortes de leurs ressources non-monétaires et de leurs contraintes de choix.

Keywords. Rationality, GARP, Non-parametric tests, Shadow prices.

Classification JEL. C14, D11, D12

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1. Introduction

Non-parametric tests applied to aggregate time series data confirm, in general, the hypothesis of a representative agent maximizing utility function. Neither Varian (1982) analyzing aggregate consumption data, nor Swofford and Whitney (1986) who analyze quarterly data on monetary assets in the USA for the period 1969:1-1979:4, found a violation[1] of the GARP axiom (Generalized Axiom of Revealed Preference). Chalfant and Alston (1988), analyzing US meat demand from 1947 to 1983 and Australian quarterly meat demand from 1961:1 to 1984:4, test the consistency of the data with an axiom called SARP (Strong Axiom of Revealed Preference) which is more restrictive than GARP. They find no violation for the US and only two violations for Australia. The reason for such a result over aggregate times series data are now well-known : when using aggregate data, few or no budget hyperplans intersect, leading to *a trivial respect* of GARP.

On the contrary, non-parametric tests performed over micro-economic data do not give a clear cut answer about the issue of the respect by consumers of rationality axioms (GARP, SARP, WARP). For instance, Mattei (1994) using a monthly Swiss Consumer Panel Data (1975-89)[2] shows that half the households and all households, whose choices have been recorded during at least 60 months, violate GARP. Especially the 19 households whose choices have been recorded from 1975 to 1989 violate GARP. Sippel (1997, page 1443) over two experimental consumer data sets of respectively 12 and 30 individuals, concludes the following: *"We find a considerable number of violations of the revealed preference axioms, which contradicts the neo-classical theory of the consumer maximising utility subject to budget constraint. We should therefore pay closer attention to the limits of this theory as a description of how people actually behave, i.e. as a positive theory of consumer behaviour".* The author indeed finds out in his first experiment, 11 individuals (over a total of 12) who violate SARP and 5 individuals (41.7%) who violate GARP. He also finds out in his second experiment, 22 individuals (over a total of 30) who violate SARP and 19 who violate GARP. However an original[3] paper by Février and Visser (2000) over two experimental data sets of respectively 60 and 60 individuals finds out "only" 35 (over 120) individuals (29%) who violate GARP. Moreover Famulari (1995) using repeated cross-sections of households from the 1982-1985 Consumer Expenditure Survey[4] divided into mutually exclusive 43 demographic groups according to eight demographic characteristics finds out that 42 households groups respect GARP.

While it is easy to compare the works of Sippel (1997) and Février and Visser (2000), because they both use experimental data, it is not easy to compare the work of Mattei (1994) and the one of Famulari (1995). The first difference concerns the nature of the data. Famulari uses a micro-economic

[1]However some violations are found over some subsets of the entire data, simply indicating general interaction between monetary assets.
[2]It includes actually several overlapped panel data over the period since only 19 households remain from 1975 to 1989.
[3]The subjects are not, like usual, recruited from the university staff or students, but are randomly selected from DIJON (France) population. Moreover the way the tests were performed is very original.

data and groups the households in order to do her tests. Hence the individuals she considers are in fact groups of households, while Mattei uses a panel data. The second difference is a methodological one. To decide whether an household violate an axiom, Famulari uses the concept of violation rate that can be found in Varian (1990). In order to understand this concept, let us recall that non-parametric tests are non-probabilistic therefore we should reject the null hypothesis (that is, the agent respects the axiom) if we find only one violation of the axiom. For instance in Mattei's study, a household from 1975 to 1989 faced 180 different bundles and the maximal number of comparisons is 16110 (C^2_{180}). However we should conclude that the agent violates GARP if we find only one violation over 16110. A violation rate of an axiom (for instance GARP) is the number of violation divided by the number of pairs belonging to the revealed preference relation. Varian (1990) has proposed the usual 5% as cut-off. Hence the 42 groups of households, which respect GARP in Famulari's study, do not have a number of violation (of GARP) equal to zero but rather a violation rate less than 5%.

The purpose of this paper is twofold. First we ask a question about the theoretical content of the non-parametric tests: does violation of a rationality axiom (SARP, GARP or WARP) necessarily mean an irrational behavior from the individual? The answer is no. On the one hand, because rationality in the basic sense of choice function theory (Richter 1971) from which demand theory is derived, only requires choices to coincide with the elements optimal with respect to the agent's preference. Hence the preference relation of an agent rational in the sense of Richter does not need to be transitive. Such an agent with non-transitive preference will however violate SARP and GARP.
On the other hand, an agent who is a utility function maximizer and changes her preference over the period covered by the test, will violate SARP, GARP and WARP. Such a violation of rationality axioms is only apparent and simply expresses preferences changing, not a true irrationality.
The second purpose is to test the axioms over an interesting panel data (Polish Consumer Panel Data 1987-1990) concerning a period with very high price variations and less real income variations. Thus bugdet hyperplans' intersections are likely to occur, so are violations of the axioms. We find out that all the 3630 households respect the rationality axioms and the 240 households who apparently violate them, actually change their preferences over the period which includes a shift from a centrally planned economy to a market oriented one. We explain these preferences changing using an econometric model, by a change of the agents' shadow prices through drastic changes of their non-monetary resources and the arise of new constraints.

The paper includes 6 sections. Section 2 introduces the non-parametric tests. We make in this section a typology of behaviors when performing a simultaneous test of SARP, GARP and WARP. Section 3 is devoted to the Polish Consumer Panel Data (1987-90). Section 4 deals with the non-

[4]Annual household expenditures are aggregated in nine categories and prices are differentiated according to the metropolitan area.

parametric tests' results and in section 5 we estimate a parametric econometric model of complete prices to explain the 240 households who apparently violate the axioms but actually change their preferences. Section 6 discusses the main conclusions.

2. The non-parametric tests

2.1 Rationale

In this paper, we use various non-parametric tests (tests of SARP, GARP and WARP) of demand theory. We test for instance the GARP axiom (Generalized Axiom of Revealed Preference) which is a necessary and sufficient condition for the existence of a utility function (concave, continuous, differentiable, and satisfying the insatiability hypothesis) maximizing the choices revealed by the data (Varian 1982). The test is called non-parametric, as opposed to parametric tests which are built by the estimation of the parameters of a pre-specified utility function. Thus the parametric tests of utility function maximizing are joint tests: *the utility function has a specific functional form and agents maximize this function.* So it is difficult to know whether the rejection of the null hypothesis is due to the specification of the utility function or to the utility function maximization hypothesis. In non-parametric test, there is of course no need to specify the utility function.

The theoretical works on non-parametric tests began with Afriat (1967). Diewert (1973), and Diewert and Parkan (1985) made essential contributions. However, from a practical point of view, Varian (1982) built the first "efficient" non-parametric test algorithm.

2.2 Method

Let D be a data set including prices P_i (vectors of R^n_+) and bundles of goods X_i (vectors of R^n_+) purchased at price P_i. Let us set the binary relation denoted R, called *revealed preference*, and defined by : $X_i \text{ R } X_j \Leftrightarrow X_iP_i \geq X_jP_i$. The interpretation is that the bundle X_i is revealed preferred to the bundle X_j if the bundle X_i is chosen while bundle X_j could have been chosen (X_i has a total expenditure evaluated at price P_i superior to the total expenditure of bundle X_j evaluated at price P_i). One defines also the following relation RS (called *strict revealed preference*) : $X_i \text{ RS } X_j \Leftrightarrow X_iP_i > X_jP_i$, where the bundle X_i is available at price P_i .

Let TR be the transitive closure of R, defined by : $X_i \text{ TR } X_j \Leftrightarrow X_i \text{ R } X_s \text{ R } ......\text{R } X_j$.

The **SARP** axiom (Ville 1946, Houthakker 1950) requires TR to be antisymmetric[5] : X_i TR X_j, $i \neq j \Rightarrow$ not(X_j TR X_i) while the **GARP** axiom requires the bilateral asymmetry of TR and RS : X_i TR $X_j \Rightarrow$ not(X_j RS X_i). **WARP** requires the revealed preference relation R to be antisymmetric: X_i R X_j, $i \neq j \Rightarrow$ not(X_j R X_i).

The empirical tests of the axioms are quite easy. For instance in the case of GARP, we perform the following procedure:

1. We build the revealed preference relation R.
2. We build the strict revealed preference RS.
3. We build the transitive closure TR of R by an algorithm of least cost path called the *Dijkstra algorithm*.
4. For each couple (X_i, X_j) belonging to TR, (i.e. such that X_i TR X_j), we test if (X_j, X_i) belongs to RS, that is to say if Xj RS X_i. If yes then this couple violates GARP and we add 1 to the number of GARP violations.

It is obvious that SARP implies GARP and WARP. However there is no relationship between GARP and WARP except in two-dimensional Euclidean space where the three axioms are equivalent (Samuelson 1948, Rose 1958). Let us now set that for finite cases, the below axioms and implications are equivalent (see Samuelson 1948, Afriat 1967, Shafer 1975).

Axiom	Implications
SARP	Stable **linear** *(transitive, antisymmetric and complete)* preference, there exists a utility function that rationalizes the data, there exists a demand function that rationalizes the data.
GARP	Stable **preorder** *(transitive and complete)* preference, there exists a utility function that rationalizes the data, there exists a demand correspondence that rationalizes the data.
WARP	Stable complete and antisymmetric preference, there exists a function that rationalizes the data (but nothing can be said about the nature of this function).

It is easy to see that the test of one of the above three axioms is not enough to give us an information about the rationality of an agent's behavior. For instance the violation of GARP over a data set may be due to, *either the violation of the optimizing behavior* (the agent does not choose the

[5]If a good X_i is preferred (in the sense of TR) to a good X_j, then X_j cannot be preferred to X_i.

best element with respect to her preference relation), *either the violation of the stability of preference* over the period (the agent changes her preference) or *the violation of the transitivity and completeness of preference axioms*. Let us take an agent who is a utility function maximizer (preorder preference maximizer) but who changes her preference over the period covered by the data set. Then the test will conclude that she violates GARP over this period (indeed the displayed preference relation is not her true preference), despite the fact that she is obviously rational.

At this stage, let us recall that the basic definition of rational choice set by Richter (1971) in choice function theory[6] does not impose any constraint over the structure of the revealed preference (in particular, it does not impose *a priori* the transitivity of the revealed preference relation). An agent is rational if she chooses the elements optimal with respect to her preferences.

Let us now take an agent with stable preference over a period and who respects the optimization principle (that is the agent is rational with respect to the basic definition of Richter, 1971). Suppose that there exist different x_1 , x_2 and x_3 , such that $x_1\ P_R\ x_2$, $x_2\ P_R\ x_3$ and $x_1\ I_R\ x_3$, where R is the agent's revealed preference relation, P_R and I_R are respectively the asymmetric and symmetric components[7] of R. Then SARP, GARP and WARP will be violated, leading to the false conclusion that the agent is not rational.

It is therefore important to be careful about the conclusions we derive from the tests and not take for granted that a violation of an axiom necessarily means an irrational behavior. We argue thereafter that the simultaneous tests of SARP, GARP and WARP can improve, a little bit, our understanding of an agent behavior over a given period.

2.3 Simultaneous Tests of SARP, GARP and WARP : A Typology of Behaviors

First we make the test of SARP. Two cases occur :

1. The agent respects SARP then she maximizes a stable linear preference over the period.
2. The agent does not respect SARP then we start two tests :
    - 2.1. Test of GARP : two cases occur.
        - 2.1.1. The agent fulfills GARP then she maximizes a stable preorder preference over the period. Since the agent violates SARP, the symmetric part of her preference relation (reflexivities excluded) is not empty.
        - 2.1.2. The agent does not respect GARP then two cases occur:

[6]Demand theory is derived from choice function theory.

[7]$x\ P_R\ y \Leftrightarrow x\ R\ y$ and not $(y\ R\ x)$; $x\ I_R\ y \Leftrightarrow x\ R\ y$ and $(y\ R\ x)$.

2.1.2.1. The set of SARP violating couples is equal[8] to the set of GARP violating couples, then assuming[9] that the agent fulfills the optimizing condition leads[10] to the conclusion that the agent respects SARP, GARP and WARP but has changed her preference at least once over the period: the apparent violations of SARP and GARP (and WARP) are due to preference instability, not to preference structure.

2.1.2.2. The set of SARP violating couples is not equal to the set of GARP violating couples, then the agent both violates GARP and SARP. However even if we assume rationality in the sense of Richter (optimizing condition), violation of GARP and SARP may simply be apparent (that is violations are due to instability of preference)[11] or may be a true one (violations are due to preference structure i.e. preference is not transitive).

2.2. Test of WARP : two cases occur.

2.2.1. The agent fulfills WARP then she maximizes a stable complete and antisymmetric preference over the period. However since the agent violates SARP, this preference relation is not transitive. This agent is rational despite the fact her preference is not transitive. She maximizes other types of "utility" functions called *variable interval functions* f(x,y) = u(x) + s(x,y) where u is a *standard utility function* and s is a *threshold function* defined from the set of objects to R^{+} (see Diaye 1999).

2.2.2. The agent does not fulfill WARP then two cases occur:

2.2.2.1. The set of SARP violating couples is equal to the set of WARP violating couples, then we get the same conclusion as in 2.1.2.1.

2.2.2.2. The set of SARP violating couples is not equal to the set of WARP violating couples, then the agent both violates SARP and WARP. Like in case 2.1.2.2, even if we assume rationality in the sense of Richter (optimizing condition), violation of WARP and SARP may be[12] a true one (violations are due to preference structure i.e. preference is not transitive) or may simply be apparent (that is, violations are due to instability of preference).

[8]Let us recall that since SARP implies GARP then the set of GARP violating couples is included in the set of SARP violating couples.
[9]There is a large consensus, among the theorists, about this assumption (see for instance Russell and Tengesdal 2001).
[10]Proof in Appendix I.
[11]Counter-example in Appendix II.
[12]Counter-example in Appendix II.

2.4 Power of the non-parametric tests

Non-parametric tests are non-probabilistic and their power is unknown. The first theoretical answer to this issue consists in trying to give a probabilistic background to the non-parametric tests (Varian 1985, Epstein and Yatchew 1985). Other methods try both to take into account the non-probabilistic dimension of the non-parametric tests and to improve their power, have been developed. We can mention the method we call *Power in the sense of Bronars*. This method developed by Bronars (1987) (see also Aizcorbe 1991) compares the results of the non-parametric tests over a given data set with the results obtained by non-parametric tests over a data set where the agents choose randomly in the sense of Becker (that is they choose their consumption bundles randomly with respect to a uniform law).

Moreover two main questions arise concerning the data: does the consistency I find, a true one ? and does the violation I find, a true violation ? Indeed some couples of bundles trivially respect GARP (or SARP, or WARP) and other trivially violates it.

A couple (X_i, X_j) *trivially respects* GARP when the total expenditure of X_i evaluated at constant price is sharply "greater" than the one of X_j evaluated at the same constant price. Indeed, we will have $X_iP_i > X_jP_i$ and $X_iP_j > X_jP_j$. The couple (X_i, X_j) *trivially violates* GARP if the total expenditure of X_i and X_j evaluated at the available price P_i is "similar". The consumer will therefore consider these two bundles as identical. This leads to $X_iP_i \geq X_jP_i$ and $X_jP_j > X_iP_j$.

Concerning the trivially consistent couples, we will use a method developed by Famulari (1995). She chooses a reference price P_o and evaluates all the consumption vectors at this price. The total expenditure of a vector X_i evaluated at this constant price P_o , will be denoted $M_i = P_o.X_i$. If a bundle X_i is much more expensive (in the sense of M_i) than a bundle X_j, it will probably be preferred, therefore taking into account the comparison of these two bundles will arbitrary increase the number of couples consistent with GARP. According to Famulari, the couple (X_i , X_j) worthy respects GARP if the following index $I = 2\times(M_i - M_j)/(M_i + M_j)$ is less than a threshold K arbitrary defined by the experimenter. Like her, we will take four different levels for K: 5%, 10%, 15% and 20%. The interpretation is the following, if K = x%, it means that we will take into account only the couples whose total expenditures evaluated at constant price P_o are different by at most x%.

Concerning the trivial cases of GARP violation, we will use a method by Afriat (1967) who proposes to use for the experiments, the relation R_ε , sub-relation of R, defined by : $X_i \, R_\varepsilon \, X_j \Leftrightarrow \varepsilon\times X_iP_i \geq X_jP_i$, where $\varepsilon \in \,]0,1]$. The parameter ε is known under the name of *Afriat Efficiency Index*

(Varian 1990). Some generalizations of this concept can be found, in Varian (1990) who allows ε to be different with respect to the bundles (the parameters ε are therefore called *Afriat generalized indexes*), in Gross (1990) or in Famulari (1995). Other methods can be found in Gross (1995a, 1995b) or Blundell *et al.* (1997).

3. The Data

Household budget surveys have been conducted in Poland for many years. In the analyzed period, the annual total sample size was about 30 thousand households, which represent approximately 0.3% of all households in Poland. The data were collected by a rotation method on a quarterly basis. The master sample consists of households and persons living in randomly selected dwellings. To generate it, a two stage, and in the second stage, two phase sampling procedure was used. The full description of the master sample generating procedure is given by Kordos and Kubiczek (1991).

Master samples for each year contain data from four different sub-samples. Two sub-samples started to be surveyed in 1986 and ended the four years survey period in 1989. They were replaced by new sub-samples in 1990. Another two sub-samples of the same size were started in 1987 and followed through 1990. Over this four-year period on every annual sub-sample it is possible to identify households participating in the surveys during all four years. The checked and tested number of households is 3736. However 3630 households remain in the data set after deleting households with missing values. The available information is as detailed as for the cross-sectional surveys: all typical socio-economic characteristics of households and individuals are present, as well as details on income and expenditures. Household expenditures are aggregated into nine categories: *Food, Alcohol and Tobacco, Culture, Energy, Clothes, Housing, Medical Care and Hygiene, Transportation and Communication, "Other".*

Finally, a large part of this panel containing demographic and income variables is included into comparable international database of panels in the frame of PACO project (Luxembourg) and is publicly available.

Prices and price indices are those reported by the Polish Statistical Office (GUS) for main expenditure items. They are quarterly observed and differentiated by 4 social categories: *workers, retired, farmers, and dual activity persons (farmers and workers).* This distinction covers implicitly the geographical differentiation: workers and retired live mostly in large and average size cities, farmers live in the countryside and dual activity persons live mostly in the countryside and in small towns. The individual price index differences can be interpreted as a consequence of the behavioral specificity due to the social class, life cycle and consumption needs. The regional differentiation is less significant in the context of panel’s period covering the time of administrated prices, identical for the whole country (1987-1989). Only the coexisting free (or parallel) markets might have created an influence on regional average price differences. These differences are taken into account through social differentiation.

Tables 3.1 and 3.2 in Appendix III present descriptive information on the Polish data. The period 1987-1990 covered by the Polish panel is unusual even in Polish economic history. It represents the shift from a centrally planned, rationed economy (1987) to a relatively unconstrained fully liberal market economy (1990). GDP grew by 4.1% between 1987 and 1988, but fell by .2% between 1988 and 1989 and by 11.6% between 1989 and 1990. Price increases across these pairs of years were 60.2%, 251.1% and 585.7%, respectively. Thus, the transitory years 1988 and 1989 produced a period of a very high inflation and a mixture of free-market, shadow and administrated economy.

4. The Results of the Non-parametric Tests

4.1. Tests of SARP, GARP and WARP

The main result is that for any household who violates one of the three axioms, the set of SARP violating couples is equal to the set of GARP violating couples and to the set of WARP violating couples. Moreover, over the 3630 households, 3390 respect the axioms and only 240 violate the three axioms and the average number of violation (among the 240 households) is 2.295. However the violations of SARP, GARP and WARP by these 240 households are due to preference change (see point 2.1.2.1 of our discussion in section 2.2). *Hence the 3630 households respect the rationality axioms. The 240, which apparently violate the axioms, in fact respect them but change their preference at least once over the period.*

Table 1.
Simultaneous tests of SARP, GARP and WARP.

	Respect of SARP, GARP and WARP : Nbviol (number of violation) = 0	**Non Respect of SARP, GARP and WARP :** Nbviol ≠ 0
Number of households	3390	240
Means of Nbviol	0	2.295
Std-error of Nbviol	0	0.928
Min. of Nbviol	0	2
Max. of Nbviol	0	9

4.2. Robustness of the Tests

Let us discuss now the robustness of our results and let us first discuss about the robustness concerning the 3390 households, which respect the three axioms. Like we have said above in section 2.4, a couple (X_i , X_j) trivially respects the axioms when the total expenditure of X_i evaluated at constant price is sharply "greater" than the one of X_j evaluated at the same constant price. When taking into account only the couples whose total expenditures evaluated at constant price P_0 are different by at most K = 20%, 94.2% of the initial 3390 households remains consistent. For K = 15%, 10% and 5%, the figures are respectively 83.5%, 63.4% and 33.5%. The results concerning the households fulfilling the axioms look robust since even at K = 5%, a large proportion of households is still consistent with the axioms. Moreover a logistic regression (see Table 3.3 in Appendix III) shows that the more the variation in absolute value of real expenditure per UC, the more the probability of violating the axioms. Thus the increase of real expenditure is not, *ceteris paribus*, a cause of respecting the axioms.

Table 2.

Simultaneous tests of SARP, GARP and WARP
when removing from the revealed preference relation R the couples (X_i, X_j) such that the total expenditures X_i of and X_j calculated at constant price are different by at most K%.

	Respect of SARP, GARP and WARP : Nbviol = 0
K=20%	3194 *households*
K=15%	2832
K=10%	2149
K=5%	1139

Let us now discuss about the robustness of the 240 households, which violate the axioms. Table 3 below gives the results of the Afriat Efficiency test. At ε = 0.99, 54.6% of the 240 households still violate the axioms but at ε = 0.87, none of them violate. However at ε = 0.87 almost all the 3630 "fictive" agents we have built in the 10 random samples, also fulfill the axioms (see table 5 below).

Table 3.

Afriat Efficiency Tests over the 240 households violating SARP, GARP and WARP.

	Non Respect of SARP, GARP and WARP : Nbviol ≠ 0
ε = 1.00	240 *households*
ε = 0.99	131 *households*
ε = 0.98	84
ε = 0.97	48
ε = 0.96	30
ε = 0.95	18
ε = 0.94	12
ε = 0.93	8
ε = 0.92	6
ε = 0.91	3
ε = 0.90	2
ε = 0.89	1
ε = 0.88	1
ε = 0.87	0

Finally the power of the test in the sense of Bronars seems good since the number of SARP violating households over 10 random samples increases of 155.5% (613.3 households on average over the 10 random samples, see table 4 below). Each random sample includes four surveys: 1987, 1988, 1989, 1990. For any survey t, we build ten data sets $D_n = \left(\tilde{X}_i, P_i\right)_{i=1\,to\,3630}$, n = 1 to 10, in the following way : *for any agent i*,

- $\tilde{X}_i = \left(\tilde{x}_i^1, \tilde{x}_i^2, \ldots. \tilde{x}_i^9\right)$ such that $\tilde{x}_i^j$ (j = 1 to 9 categories of goods) is the realization of a random variable that follows a uniform law $\left[a^j, b^j\right]$ with $a^j = \underset{i=1\,to\,3630}{Min}\, x_i^j$ and $a^j = \underset{i=1\,to\,3630}{Max}\, x_i^j$, with x_i^j = observed quantity for agent i and category j.
- $\tilde{X}_i . P_i = X_i . P_i$: random consumption bundle $\tilde{X}_i$ and observed consumption bundle X_i have the same expenditure evaluated at P_i the observed price.

Table 4.

Simultaneous tests of SARP over a 10 random samples in the sense of Becker (standard-error in parenthesis)

	Non-Respect of SARP : Nbviol $\neq$ 0
Number of households (means)	613.3 (20.14)
Means of Nbviol	2.63 (1.63)
Min. of Nbviol	2
Max. of Nbviol	12

Table 5.

Afriat Efficiency Tests over the households violating SARP over the 10 random samples.

	Sample I	**II**	**III**	**IV**	**V**	**VI**	**VII**	**VIII**	**IX**	**X**
ε = 1	590	611	617	590	615	648	643	612	615	592
ε = 0.99	420	457	440	428	453	472	464	431	462	439
ε = 0.98	318	335	306	305	325	336	341	311	322	308
ε = 0.94	70	87	83	89	102	95	100	84	88	73
ε = 0.90	16	15	22	20	22	21	23	13	19	15
ε = 0.87	5	6	4	7	6	4	3	4	1	4
ε = 0.86	4	3	2	5	4	1	2	3	1	2
ε = 0.85	2	2	1	3	2	1	2	1	0	0
ε = 0.84	1	1	0	0	1	0	0	1	0	0
ε = 0.83	1	0	0	0	1	0	0	0	0	0
ε = 0.82	1	0	0	0	0	0	0	0	0	0
ε = 0.81	0	0	0	0	0	0	0	0	0	0

4.3. Characteristics of the violating households

A Logit estimation on the violations (Table 3.3 in Appendix III) shows that no classic socio-economic characteristics such as the household's demographic structure and location, the head's age, profession or education and the survey's quarter is significant (at 5%) to predict violations[13]. The retired people are the only social category for which a significant increase in violation probability is observed.

On the other hand the violations are strongly related to the changes in the household's economic and demographic situation: there is a positive and significant relationship between the probability of axiom violation and the change in real per consumer unit total expenditure in observed periods between 1987 and 1990. The effect is the strongest and the most significant for the last period (1989-90), which coincides with the economic transition shock in Poland. Thus both factors contribute in the violation process: the way the change in individual well being is taken into account in individual's behavior and the impact of structural changes in the whole economy. In the period 1989-1990 a dramatic loss in the purchasing power is accompanied by a shift from a rationing situation on the consumption goods market to an almost equilibrium situation. The other significant effect is the change of family size (especially again in the period 1989-1990), which increases the probability of violation.

The violating and non-violating households may not differ with respect to their average characteristics (as suggested by the logit estimation) but may behave differently when facing a change in income, relative prices or other determinants of their consumption. To check this, we estimate a Quadratic Almost Ideal Demand System over the pooled surveys.

The estimation of the Quadratic Almost Ideal Demand System QUAIDS was made by the convergence algorithm proposed by Banks *et al.* (1997) :

$$(4.3.1)\quad w_{iht} = \alpha_i + \Sigma_j\, \gamma_{ij} \ln p_{jt} + \beta_i \ln [m_{ht}/a(p_t)] + \{[\lambda_i/b(p_t)] \ln [m_{ht}/a(p_t)]\}^2 + W_{ht}.\gamma_i + u_{iht}$$

$$\text{with } \ln a(p_t) = \alpha_0 + \Sigma_i\, \alpha_i \ln p_{it} + 0.5\, \Sigma_i \Sigma_j\, \gamma_{ij} \ln p_{it}.\ln p_{jt} \quad \text{and} \quad b(p_t) = \Pi_i\, p_{it}^{\beta i}$$

where w_{iht} is the budget share for good i, individual h and period t, p_{it} the price of good i at period t, m_{ht} the total expenditure of individual h at period t. As the estimated parameters α_i, β_i, γ_{ij} enter non-linearly into the equation, a *first step* consists to estimate equation (4.3.1) using a Stone price index $a(p_t) = \Pi_i\, p_{it}^{wi}$ with w_i the average budget share of good i over individuals and periods (that is, imposing $\alpha_0 = \gamma_{ij} = 0$ and $\alpha_i = w_i$). Price elasticities are corrected to take into account the difference between the exact price index $a(p_t)$ and the Stone index, as described by Pashardes (1990). In the

[13]This result remains true when restricting the estimation to those households for which the expenditure evaluated at constant price change between two surveys is greater than 20% (there remain 3263 households), 15%, 10% or 5%.

second step, the estimated γ_{ij} and β_i are used to compute $b(p_t)$. At each step, $b(p_t)$ is updated and the system is linear in parameters.

Blundell and Robin (1999) prove the consistency and asymptotic efficiency of this iterative procedure to the maximum likelihood estimate. The estimation is made under additivity, and homogeneity restrictions, despite homogeneity is not accepted by the data except for clothing (results are similar when homogeneity is not constrained). The cross-section and time-series parameters are estimated by pooling the four surveys with quarter and period dummies to take into account all institutional changes. Cross-section parameters are estimated on pooled cross-section (third iteration) and the time-series parameters on first differences (third iteration) because the usual between and within transforms do not converge[14].

We find (Table 3.4 in Appendix III) that income elasticities are significantly greater for the violating households concerning Food at Home, and smaller concerning dynamic expenditures such as Food away from Home. These differences are not significant for cross-section estimates, because of the endogeneity bias due to the correlation between the specific effects and the relative income. They are, on the contrary, significant for the first difference estimates, which cancel this endogeneity bias. Thus probably, either the income distribution and the structure of income changes (for instance as far as the transitory and permanent changes are concerned) differ between the two populations, or the food expenditures of the violating households are more sensitive to income changes (since these violating households are subject to a subsistence constraint during this period[15]).

As far as the level of well being is concerned, the violating households have a smaller total expenditure than the non-violating, by 3.7%, 3.1% and 7.4% in the first three years. Compared to a prediction (for the whole population) by variables such as *age*, *family composition*, *location*, *profession and education*, their total expenditures are smaller by 2.5 to 4.9% between the first three years, and greater by 3.5 in 1990[16]. Moreover, both income and total expenditures increase less for the violating households between 1988 and 1989 (+23.0% versus +32.3% for disposable income per unit of consumption, +2.6% versus +6.7% for total expenditures) but decrease much less between 1989 and 90 (-32.7% versus -39.0, -21.3 versus -34.1). Similar variations are observed for low-income families. Thus, the violating households seem to have a smaller level of well being, at least between 1987 and 89, and to have been confronted to a different pattern of variation of their well being during this period.

[14]Probably because of the high degree of non-linearity over the integrability terms b(p). Moreover the estimation on first differences eliminates the endogeneity bias by canceling the specific effects between two periods (recall that the within transform only cancels the specific effect over the whole period).

[15]Despite their smaller level of well-being, their budget share for food consumption is not systematically greater than the budget share of the non-violating households or than the budget share predicted by the quadratic demand system. Note that, as their income parameters differ from those of the non-violating, this estimation is actually misspecified.

[16]Including income to predict total expenditures does not change this pattern.

That leads to the question whether the violating households were specifically confronted (compared to the non-violating) to some constraints such as subsistence constraints for the poor in particular.

To estimate the number of poor, first, we compute the proportion of households having less than the average income minus its standard error. The rich are defined as households having more than the average income plus its standard error. The violating population contains much more “rich” than the non-violating (21.7% versus 16.3% in 1987) and a little more “poor”. Actually the income distribution of the violating population is more polarized: the income variation coefficient is 12.3 for the violating versus 9.5 for the non-violating. Second, we define the poor using a composite index matching three criteria: an income smaller than the first quintile of the whole population, total expenditure smaller (by 25% in Table 3.5) than the average expenditure for the reference population (defined by age, education, location, socio-professional category and family structure), and food budget share greater than 25% of the average share for this reference population (see Gardes *et al.* 2000, for a discussion of this definition[17]), the violating contains more rich and a bit more poor than the non-violating. More important, one half of the households switches to another social class (for instance from the poor class to the quasi-poor) during the whole period, especially between 1988 and 1989, both for the violating and the non-violating (see Tables 3.5 and 3.6). This indicates that these violating households have been confronted to new conditions of living and new constraints during the whole period.

Thus, the violation process may be explained as a coherent response to the changing economic position of the household.

The next step of this paper is to build a model of subsistence constraints and shadow prices, allowing explaining theoretically apparent irrationalities among poor households with a strong decrease in income.

5. Subsistence Constraints and Shadow Prices.

5.1 Rationale

Polish households were confronted to drastic subsistence constraints during these four years. Indeed, their level of well-being was very influenced by inflation and macroeconomic shocks: between 1988 and 1989, the real income per U.C. for households belonging to the first quartile decreased by 30% and increased by the same amount for households belonging to the last quartile. The inverse change occurred between 1989-90. Such constraints correspond to shadow price effects, which may

[17]Four other social classes are defined by the same variables. The results are robust to different definitions of the limits.

strongly influence household's consumption and saving decisions. In such a situation, monetary prices no longer explain alone the household choices.

Suppose that *monetary price* and *two shadow prices* corresponding respectively to *non-monetary resources* and to *constraints faced by the households* are combined together into a complete price. Expressed in logarithm form, we have:

$p_c = p_m + p_{nm} + p_{cs}$ where p_c denotes the (logarithm of the) complete price, p_m denotes the (logarithm of the) monetary price, p_{nm} denotes the (logarithm of the) shadow price corresponding to non-monetary resources and p_{cs} denotes the (logarithm of the) shadow price corresponding to constraints faced by the households. However it is not possible with our data to distinguish between the two components of the shadow price $\pi = p_{nm} + p_{cs}$.

We want to test the assumption that the 240 violating households faced during this period a change in their shadow prices through drastic modifications of their non-monetary resources (such as a substitution between the monetary and non-monetary component of price due to the existence of queuing) and the appearance of new constraints (such as subsistence constraints which increase the non-monetary prices of the constrained goods).

5.2 Measuring the Shadow Prices

Suppose that two estimations of the same equation : $x_{iht} = Z_{ht}.\beta_i + u_{iht}$, for good i (i = 1 to n), household h (with h = 1 to H) in period t (t = 1 to T), with $Z_{ht} = (Z_{1ht}, Z_{2ht})$, are made on cross-section and time-series over the same data-set. Let us set $u_{iht} = \alpha_{ih} + \varepsilon_{iht}$ where α_{ih} is the specific effect which contains all permanent components of the residual for individual h and good i. As discussed by Mundlak (1970), the cross-section estimates can be biased by a correlation between the explanatory variables Z_{1ht} and this specific effect. Such a correlation is due to latent permanent variables (such as an event during the infancy, characteristics of parents or permanent wealth). This correlation comes from the relationship existing between such latent variables and some explanatory variables Z_{1ht} in the cross-section : for instance, the relative income position of the household can be related to its wealth or to its inheritance. Those effects are embedded in the specific effect. Thus, the correlation δ_i between the time average of the vector of the explanatory variables, $Z_{1ht} = (z^k_{1ht})_{k=1 \text{ to } K1}$, transformed by the Between matrix: $BZ_{1ht} = \{(1/T) \sum_t z^k_{1ht}\}_{k=1 \text{ to } K1}$, and the specific effect α_{ih} , $\alpha_{ih} = BZ_{1ht}.\delta_i + \eta_{ih}$, will add to the parameter β_i of these variables in the time average estimation : $Bx_{iht} = BZ_{1ht}.(\beta_i + \delta_i) + \eta_{ih} + B\varepsilon_{iht}$, so that the between estimates are biased. The difference between the cross-section and the time-series estimates amounts to δ_i.

Let us now assume that the shadow price π_{iht} of good i for household h in period t, depends on a vector of *(endogeneous) characteristics*, Z_{1ht}, and on a vector of *(exogeneous) characteristics* Z_{2ht}, that is $\pi_{iht} = f_i(Z_{1ht}, Z_{2ht})$ *(equation 5.2.1)*. Let us assume also that the consumption function for good i is $x_{iht} = g_i(p_{ht}, Z_{1ht}, Z_{2ht}, S_{ht})$ *(equation 5.2.2)* with p_{ht} the vector of prices p_{jht} containing (if it exists) a shadow, unknown component π_{jht}, and S_{ht} the vector of all other determinants.

From the shadow price function $\pi_{iht} = f_i(Z_{1ht}, Z_{2ht}) = Z_{1ht}.\theta_1 + Z_{2ht}.\theta_2 + \lambda_{ih} + \mu_{iht}$, we define a vector of *endogeneous variables* Z_{1ht} as a vector of all variables correlated with the specific effect λ_{ih} . For instance, the relative income position of the household, supposed to be invariant, can determine its location, which is correlated with purchase constraints. Thus the household's relative income may be related to permanent components of the shadow prices corresponding to these constraints. The vector of *exogeneous variables* Z_{2ht} is therefore a determinant of the shadow price, which is not correlated with λ_{ih}.

If the shadow prices are unknown and do not appear in the consumption function 5.2.2, their influence are included into the residual, so that the specific effect α_{ih} of the consumption function (with a residual $u_{iht} = \alpha_{ih} + \varepsilon_{iht}$) contains λ_{ih}. Therefore, the coefficients of the endogeneous variables Z_{1ht} in the consumption function are subject to an endogeneity bias, which makes their cross-section, and time-series estimates different, while the exogeneous variables have coefficients, which do not differ between the two dimensions. Thus, the part of the shadow prices, which is explained by Z1ht, can be recovered by the endogeneity bias (in the consumption function), as revealed by the difference between the cross-section and time-series estimates of the coefficients of Z_{1ht}.

Indeed, the marginal propensity to consume with respect to Z_{1ht}, when considering the effect of the shadow prices π_{jht} on consumption, can be written:

(5.2.3) $dx_{iht}/dZ_{1ht}= dg_i/dZ_{1ht} + \Sigma_j\ (dg_i/d\pi_{jht}).(d\pi_{jht}/dZ_{1ht})$.

The second term differs between cross-section and time-series variations because of the endogeneity bias in the shadow price propensity with respect to Z_{1ht}. So, comparing two different households surveyed in the same period, this bias adds to the direct consumption propensity with respect to Z_{1ht}, as estimated on time-series. This parameter will therefore differ between cross-section and time-series estimations. For instance, the influence of the head's age cohort or income may differ on cross-sections and time-series if the shadow prices depend on cohort effects or relative income position (note that the same can occur for monetary prices). Hence the comparison of estimations computed on cross-sections and time-series reveals the difference of the shadow price system between two households.

The component $\Sigma_j\ dg_i/d\pi_{jht}.d\pi_{jht}/dZ_{1ht}$ of the marginal propensity of endogeneous variables can be used to reveal the variation of shadow prices over Z_{1ht}, $d\pi_{jht}/dZ_{1ht}$, since it can be computed by resolving a system of n linear equations after having estimated the price marginal propensities $dg_i/d\pi_j = \gamma_{ij}$.

We consider here only the *direct effect* through the price of good i : $\gamma_{ii}.d\pi_i/dZ_1$ of the variables Z_{1ht}, so that $d\pi_i/dZ_1 = [\beta_i^{(c.s.)} - \beta_i^{(t.s.)}]/\gamma_{ii}$. The price effect γ_{ii} is supposed to be the same for monetary and shadow prices. Thus, the change between two periods in the shadow price can be written: $d\ln\pi_{iht} = \sum_k (d\pi_i/dz^k{}_1).dz^k{}_{1ht}$. Under the homogeneity assumption (of degree m) of the shadow prices over variables Z_{1ht}, the value of the shadow prices can be computed as $\ln\pi_{ih} = m\sum_k(d\pi_i/dz^k{}_1).z^k{}_{1ht}$. However, this homogeneity assumption is quite strong, and we will prefer to compute only the change in the shadow logarithmic prices. Let us call I_a and I_q respectively the average absolute and quadratic indexes for the absolute and quadratic changes in the shadow logarithmic price:

$I_a(\pi^v) = (1/H).\Sigma_h \{(1/T\text{-}1)\ \Sigma_t\ \Sigma_i\ w_i.|d\ln\pi_{iht}|\}$ and

$I_q(\pi^v) = \{(1/H).\Sigma_h\ [(1/T\text{-}1)\ \Sigma_t\ \Sigma_i\ w_i.(d\ln\pi_{iht})^2]\}^{0.5}$.

.

5.3. Results.

We compute the shadow prices by estimating a Quadratic Demand System separately for the two types of households. However since the non-violating population is much more numerous, it is not easy to compare the endogeneity biases between the two populations. Moreover, the estimation is not precise enough for the violating households (see Appendix V for details). Thus, our preferred estimation was performed on the whole population with a dummy variable indicating the violating households only for the logarithmic income and squared logarithmic income variables, which differ significantly (see section 4.3 and Table 3.4 in Appendix III). Therefore the variations of the shadow prices indicate the changes in the conditions of choice for the two populations, weighted by the differences between the same cross-section and the time-series estimates (except for income).

Table 6 and Table 4.1 (in Appendix IV) present an average of the shadow price absolute differences between two periods for the violating and non-violating populations. The price elasticities differ somewhat in various estimations. The shadow prices heavily depend on the value of the price marginal propensity to consume the good i (γ_{ii}). We calibrate it under the Frisch constraint between income and price elasticities: $E^p{}_i = -\varpi.E^y{}_i \Rightarrow \gamma_{ii} = (1-\varpi).w_i - \varpi.\beta_i$, where $E^p{}_i$ and $E^y{}_i$ are respectively the direct price-elasticity and the income-elasticity for good i, ϖ the Frisch income flexibility (calibrated as 0.5 as recommended by Selvanathan 1993, Chapter 6) and β_i the estimated coefficient of log-income in an AI Demand System estimation.

Table 6.

Index of Quadratic Changes of the Logarithmic Shadow Prices

Period	All Expenditures $I_q(\pi^v)$ NV	V	Food at Home $(d\ln\pi_{iht})^2$ NV	V	Housing $(d\ln\pi_{iht})^2$ NV	V	Clothing $(d\ln\pi_{iht})^2$ NV	V
1987-88	1.290	1.498	0.078	0.109	0.070	0.086	0.031	0.022
	(.010)	(.021)	(.005)	(.013)	(.003)	(.006)	(.001)	(.001)
1988-89	1.500	1.665	0.098	0.137	0.080	0.119	0.037	0.025
	(.006)	(.014)	(.007)	(.017)	(.003)	(.009)	(.001)	(.001)
1989-90	1.835	2.039	0.159	0.204	0.134	0.172	0.063	0.039
	(.005)	(.011)	(.015)	(.031)	(.007)	(.016)	(.002)	(.002)

Legend.

NV = Non-violating households

V = Violating households

Index[18]. $I_q(\pi^v) = \{(1/H).\Sigma_h [(1/T\text{-}1)\ \Sigma_t\ \Sigma_i\ w_i.(d\ln\pi_{iht})^2]\}^{0.5}$

Estimation. 1987-1990 under additivity and homogeneity constraints, on *Between and Within Transforms* for 1987-1990. See Table 3.4 in Appendix III for the specification on instrumented total expenditures.

Computation of the shadow prices. $d\ln\pi_{iht} = \Sigma_i\ (d\pi_{iht}\ /dZ_{1ht}).dZ_{1ht}$. The marginal propensity dp_{vi}/dZ_{1h} is computed with the price marginal propensity γ_{ii} (indexes TP2, TP2' in Table 4.1 of Appendix IV). Only changes dZ_{1h} of log-income and its square, logarithmic age of the head and the proportion of children in the family are taken into account[19].

Computation of the variances. See Appendix VI

Three main results :

(i) The variations of shadow prices are significantly different from zero[20] for almost all groups of commodities. So, these shadow prices may distort the optimal households' consumption choices.

[18]An index of absolute variations leads to similar results. Estimations on pooled cross-sections and first differences under different hypotheses upon the price coefficient used to compute the shadow prices and the differentiation of the two populations in the estimation, give also similar results (see Table 4.1 and 4.2 in Appendix IV).

[19]When adding the logarithmic relative prices to endogeneous variables Z_{1ht} supposed to contain these four determinants, one must use price parameters which depend, for cross-sections on quarter price changes and differences between social categories, and for time-series, on annual price changes. The difference between the four social categories are known only for one period, so that the cross-section and time-series estimates of the price parameters may be similar. The difference between cross-section and times-series estimates is probably due to random factors. Thus, it seems better to take only the four determinants into account.

[20] Normality being assumed for index I_q. The distribution of this index is explored by Monte Carlo in Appendix VI.

(ii) The change in the logarithmic shadow prices between two periods *increases significantly during the period*: this increase is particularly important between 1989 and 1990, when inflation, households income variation and economic institutions experienced a dramatic change. This proves the appearance or disappearance of constraints or the change in non-monetary resources, especially for the violating households.

(iii) The change in the logarithmic shadow prices appear to be greater for the violating households, by 11% to 16% for the quadratic distance in table 6 (for different specification of the consumption function and of the calibration of the price coefficient used to compute the shadow prices, the changes are even greater, see tables in Appendix IV). Table 6 shows that this phenomenon appears for almost all periods concerning Food at Home, Housing and Clothing expenditures (which sum to 74% of all expenditures). Thus, it seems overall that, between 1987 and 1990, the violating households are confronted to significantly greater changes in their environment than the non-violating.

The change in shadow prices revealed by the endogeneity bias on cross-section estimates thus seems to be important for both populations, and significantly greater for the violating households during this period. Non-rational behavior, as obtained by our non-parametric tests, may thus be due to special choice conditions which can be rationally faced by the households, but which cannot be taken into account in the estimations.

6. Conclusion

The purpose of this paper is to contribute to the current discussion on the compatibility of consumers' behaviour in "real" life with the choice consistency axioms. The computation of non-parametric tests leads to three main results. *First*, it shows theoretically that non respect of rationality axioms (SARP, GARP, WARP) does not necessarily mean a non-rational behavior. *Second*, the tests on Polish Consumer Panel Data confirm the theoretical result. They seem robust. Indeed, we are using a panel data covering a period of very high price variations and less real income changes (budget intersections are therefore more likely to occur, so are violations of the axioms). Prices are quarterly observed and differentiated by socio-professional category (the computation of quantities is therefore good), the power of the tests in the sense of Bronars is satisfying. We find that for the violating households, the set of SARP violating couples, the set of GARP violating couples and the set of WARP violating couples, coincide. A theoretical consequence is that these households only apparently violate the rationality axioms. The apparent violation is due to the fact that these households change their preferences over the period. *Third*, the logit and demand system estimates confirm the above empirical result. Indeed, the higher the decrease of real expenditure per UC, the higher the probability of violation. It suggests that apparent irrationality is due to a change in choice conditions, due itself to new constraints occurring (subsistence constraints) over the period or to the change in non-monetary resources (the disappearing of queuing). A test of an econometric model of shadow prices seems to confirm this explanation. Two consequences can be derived : *First*, it is important, when estimating a demand system, to separate the violating consumers by a non-parametric test, so that the remaining population conforms better to the theoretical assumptions of such estimations (stability, homogeneity and symmetry of price effects)[21]. *Second*, it is also important, among the violating consumers, to separate the true violating consumers, who do not change their preference over the period, from the apparent violating, who change their preferences, by comparing the separate violations of the three axioms WARP, GARP and SARP. Moreover, instead of eliminate directly the true violating households from the data set, one could estimate the household's shadow prices and use them in the estimation of the Demand System.

[21]See Février and Visser 2000, on experimental data.

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APPENDIX I

Remark. It is obvious that when the set of SARP violating couples is equal to the set of GARP violating couples then SARP⇔GARP.

Lemma 1.

1. [SARP ⇔ GARP] implies [TR ⇔ RS ⇔ R].

2. [TR ⇔ RS ⇔ R] implies [SARP ⇔ GARP ⇔ WARP]

Proof.

1. We know that RS implies R and R implies TR. By transitivity of the implication relation, we have RS implies TR. Then to show equivalence (up to reflexivities) between RS, R and TR, it is sufficient to show that TR implies RS when SARP is equivalent to GARP. Let us recall that SARP ⇔ GARP means that $[(X_i , X_j) \in TR, i \neq j \Rightarrow (X_j , X_i) \in TR^c] \Leftrightarrow [(X_i , X_j) \in TR \Rightarrow (X_j , X_i) \in RS^c]$ where TR^c and RS^c are respectively the complements of TR and RS. This obviously implies that TR coincides with RS, up to reflexivities : R = TR and R = RS + {(X, X)}.

2. The result follows obviously from the fact that TR, RS and R coincides up to reflexivities (according to condition 1 above).

■

Proof of typology's case 2.1.2.1.

According to lemma 1, R and TR coincides. Since TR is by definition transitive then R is also transitive (R is however not antisymmetric since WARP and SARP are violated). R is reflexive and transitive and can therefore be expanded in a preorder (Szpilrajn 1930). R can be "represented" by a utility function. If R is the agent's true preference over the period then since we have assumed the agent to fulfill the optimizing condition, she maximizes a utility function. Hence the agent should respect GARP (and SARP and WARP which are here equivalent). R is therefore not the agent's true preference relation and her preference has changed at least once over the period. Thus the structure of the agent's preference is not the cause of her WARP, GARP and SARP violations.

■

APPENDIX II

Counter-example in typology's case 2.1.2.2 : violation due to preference changing.

Let us restrict ourselves to the set $\{x_1, x_2, x_3, x_4\}$ and let us suppose R the revealed preference relation to be a preorder ($x_1 P_R x_2$, $x_1 P_R x_3$, $x_1 P_R x_4$, $x_2 I_R x_3 I_R x_4$), and let us suppose RS the revealed strict preference structure to have the following structure ($x_1 P_{RS} x_2$, $x_1 P_{RS} x_3$, $x_1 P_{RS} x_4$, $x_2 I_{RS} x_3$, $x_2 J_{RS} x_4$, $x_3 J_{RS} x_4$) where J_{RS} is the incomparability[22] in the sense of RS. It is easy to see that TR the transitive closure of R coincides with R. It is also easy to see that both SARP and GARP are violated and the set of GARP violation couples is included in the set of SARP violation couples. However R is a preorder and can therefore be represented by a utility function. Like in typology 2.1.2.1 we can conclude in this case that R is not the agent true preference relation, she respects the SARP and GARP and her preference is not stable over the period.

■

Counter-example in typology's case 2.2.2.2 : violation due to preference structure or to preference changing.

For simplicity, let us restrict ourselves to the set $\{x_1, x_2, x_3\}$. It is easy to see that R the revealed preference relation has necessarily the structure ($x_1 P_R x_2 P_R x_3 I_R x_1$) or ($x_1 P_R x_2 I_R x_3 I_R x_1$) if we want TR the transitive closure of R to have the following structure $x_1 I_{TR} x_2 I_{TR} x_3 I_{TR} x_1$. That leads to a violation of both SARP and WARP, and the set of WARP violation couples is included in the set of SARP violation couples. R is obviously acyclic, so the violation of SARP and WARP may be caused by the acyclic structure of R despite the fact the agent is rational in the traditional sense of Richter. However may be the acyclic structure of R is itself due to the non stability of the agent's preference over the period, R is therefore, in this latter case, not the agent's true preference.

■

[22] $x J_{RS} y \Leftrightarrow$ not (x RS y) and not (y RS x).

APPENDIX III

Table 3.1

Means and standard deviations of variables used in the Polish panel analyses

	1987	1988	1989	1990
Budget share for food	0.480 (.14)	0.461 (.15)	0.469 (.18)	0.540 (.15)
Real total per cu expenditure index (1987=100)	100	105.7	118.6	79.9
Relative food price index (pfood/ptotal)	0.961 (.013)	0.902 (.06)	0.992 (.19)	1.145 (.03)
Ln household total expenditure	10.65 (.45)	11.17 (.49)	12.25 (.79)	14.14 (.50)
Ln head's age	3.789 (.33)	3.809 (.32)	3.824 (.32)	3.842 (.32)
Consumer units number	2.500 (.98)	2.471 (.99)	2.432 (.986)	2.415 (.98)
Ln family size	1.140 (.59)	1.121 (.60)	1.095 (.61)	1.081 (.61)
Number of Households	3630	3630	3630	3630

Table 3.2
Average budget shares

Budget shares	1987	1988	1989	1990
Food	0.432	0.400	0.435	0.483
Alcohol and tobacco	0.041	0.037	0.031	0.129
Clothes	0.129	0.141	0.145	0.096
Dwelling	0.110	0.112	0.125	0.097
Energy	0.033	0.039	0.022	0.039
Health and hygiene	0.026	0.024	0.020	0.026
Transp. and communic.	0.050	0.062	0.063	0.066
Culture and entertain.	0.066	0.078	0.075	0.080
Other	0.028	0.025	0.020	0.031
Financial operations	0.087	0.081	0.057	0.050

Table 3.3
Violation Rates According to Various Socio-economic Characteristics
Logistic Function Estimates

Variable	Parameter Estimate	Standard Error	Wald Chi-Square	Pr > Chi-Square
INTERCEPT	-3.1525	0.2755	130.9421	0.0001
%Change in real per CU total expenditure (88-87)	0.2218	0.1959	1.2809	0.2577
%Change in real per CU total expenditure (89-88)	0.4830	0.2125	5.1645	0.0231
%Change in real per CU total expenditure (90-89)	0.8111	0.1843	19.3713	0.0001
Change in food budget coeff (88-87)	0.4726	0.7823	0.3649	0.5458
Change in food budget coeff (89-88)	0.8673	0.6761	1.6457	0.1996
Change in food budget coeff (90-89)	0.8082	0.6384	1.6027	0.2055
Change in family size (88-87)	-0.1307	0.1316	0.9865	0.3206
Change in family size (89-88)	-0.2744	0.1631	2.8289	0.0926
Change in family size (90-89)	0.2834	0.1103	6.6050	0.0102
Family head’s age (less than 40) (ref)				
Family head’s age (41-60)	-0.0227	0.1705	0.0178	0.8939
Family head’s age more than 60	-0.1750	0.2693	0.4224	0.5157
Location (large city) (ref)				
Location (average size city)	0.3327	0.2370	1.9697	0.1605
Location (small town)	-0.2552	0.3553	0.5158	0.4726
Location (countryside)	0.3124	0.2128	2.1555	0.1421
Education (college-university level)(ref)				
Education (average level)	-0.1140	0.1643	0.4817	0.4877
Education (primary school level)	-0.0311	0.2934	0.0112	0.9157
Wage earners (ref)				
Farmers	0.0174	0.2269	0.0059	0.9389
Mixte: farmers and wage earners	-0.3551	0.2343	2.2966	0.1297
Retired	0.3624	0.2329	2.4218	0.1197
Survey’s quarter (1) (ref)				
Survey’s quarter (2)	0.1609	0.2052	0.6154	0.4328
Survey’s quarter (3)	0.4147	0.2022	4.2074	0.0402
Survey’s quarter (4)	0.3766	0.2121	3.1531	0.0758

Number of Observations: 3630
Model Fitting Information and Testing Global Null Hypothesis BETA=0

Criterion	Intercept Only	Intercept and Covariates	Chi-Square for Covariates
AIC	1769.618	1759.969	.
SC	1775.815	1902.500	.
-2 LOG L	1767.618	1713.969	53.649 with 22 DF (p=0.0002)
Score	.	.	55.472 with 22 DF (p=0.0001)

Association of Predicted Probabilities and Observed Responses

Concordant = 63.4%	Somers' D = 0.287
Discordant = 34.7%	Gamma = 0.292
Tied = 1.9%	Tau-a = 0.035
(813600 pairs)	c = 0.643

Table 3.4
Income Elasticities (QUAIDS)

	Cross-section Elasticities		First-difference Elasticities	
	NV	V	NV	V
Food at home	0.637 (.015)	0.747 (.054)	0.774 (.018)	1.022 (.064)
Food away	1.574 (.211)	1.128 (.564)	1.574 (.253)	-0.005 (.744)
Alcohol and tobacco	1.115 (.041)	0.778 (.051)	0.976 (.048)	0.580 (.174)
Dwelling	1.311 (.044)	1.495 (.054)	1.316 (.062)	1.427 (.227)
Energy	0.358 (.071)	0.185 (.039)	0.805 (.095)	0.583 (.341)
Clothing	1.080 (.031)	1.111 (.107)	1.026 (.041)	0.861 (.148)
Transport and Communication	1.676 (.059)	1.715 (.207)	1.261 (.078)	1.144 (.277)
Health and Hygiene	0.627 (.043)	0.564 (.049)	0.681 (.057)	0.442 (.197)
Culture and Education	1.529 (.047)	1.280 (.067)	1.327 (.067)	0.921 (.232)
Other	1.377 (.071)	1.516 (.213)	1.278 (.103)	1.354 (.288)
Financial Operations	1.834 (.059)	1.398 (.212)	1.399 (.081)	0.790 (.287)

Specification: $w_{iht} = \alpha_i + \Sigma_j \gamma_{ij} \ln p_{jt} + \beta_i \ln [m_{ht}/a(p_t)] + \{[\lambda_i/b(p_t)] \ln [m/a(p)]\}^2 + W_{ht}.\gamma_i + u_{iht}$ with $\ln a(p_t) = \alpha_0 + \Sigma_j \alpha_i \ln p_{it} + 0.5 \Sigma_i\Sigma_j \gamma_{ij} \ln p_{it}. \ln p_{jt}$ and $b(p_t) = \Pi_i p_{it}^{\beta i}$
Logarithm of total Expenditures instrumented.
Other determinants : logarithmic age of the head, proportion of children in the family, relative logarithmic prices, education and location dummies, quarter dummies for each year. The true price index is approximated by a Stone price index.
Estimation : by convergence, third iteration for both dimensions, on the integrability parameter b(p). Additivity and homogeneity constrained.
Dataset : Polish panel including 3630 households for period 1987.I-1990.IV.

Table 3.5
Proportion of Social Classes (%)

k	Poor	Poor	Quasi-Poor	Quasi-Poor	Middle Class	Middle Class	Quasi-Rich	Quasi-Rich	Rich	Rich
1987										
	NV	V	NV	V	NV	V	NV	V	NV	V
O.25	143	13	918	65	1282	89	916	63	131	10
	4.22	*5.42*	*27.08*	*27.08*	*37.82*	*37.08*	*27.02*	*26.25*	*3.86*	*4.17*
0.15	223	21	701	48	1592	109	675	47	199	15
	6.58	*8.75*	*20.68*	*20.00*	*46.9*	*45.10*	*19.91*	*19.58*	*5.87*	*6.25*
0.10	266	26	563	38	1770	123	556	35	235	18
	7.85	*10.83*	*16.61*	*15.83*	*52.21*	*16.40*	*51.25*	*14.58*	*6.93*	*7.50*
1988										
0.10	*8.2*	*7.5*	*16.8*	*18.3*	*51.6*	*50.4*	*15.6*	*12.9*	*7.7*	*10.8*
0.25	*4.2*	*3.8*	*27.1*	*27.1*	*38.5*	*37.9*	*25.5*	*25.4*	*4.7*	*5.8*
1989										
0.10	*8.9*	*10.4*	*16.3*	*15.8*	*48.4*	*53.3*	*16.3*	*12.1*	*10.1*	*8.3*
0.25	*5.10*	*4.17*	*26.0*	*29.8*	*36.3*	*39.2*	*25.2*	*20.8*	*7.4*	*6.3*
1990										
0.10	*5.84*	*5.92*	*17.0*	*15.4*	*51.9*	*53.3*	*16.2*	*15.00*	*9.0*	*10.*
0.25	*2.42*	*1.25*	*27.6*	*26.7*	*37.2*	*39.2*	*27.7*	*25.8*	*5.0*	*7. 9*

Poverty criteria:

(i) Income per unit of consumption below the first quintile
(ii) Total expenditure per unit of consumption smaller than (1- k) time the average for a reference population.
(iii) budget share for food at home greater than (1+k) the average for a reference population.

Definition of social classes:
The rich are defined symmetrically, the quasi-poor (respectively the quasi-rich) as having two over the three attributes of the poor (respectively the rich) and not being rich (respectively the poor) for the third.
Number and *per-cent* of each population violating (V) or non-violating (NV).

Table 3.6
Social Class Changes (%)

k; n of classes		87/88	88/89	89/90	Average
0.10; 1	NV	0.460	0.592	0.524	0.525
0.10;1	V	0.429	0.554	0.517	0.500
0.10; >1	NV	0.079	0.168	0.112	0.120
0.10;>1	V	0.092	0.163	0.100	0.118
0.25;>1	NV	0.038	0.145	0.077	0.088
0.25;>1	V	0.042	0.133	0.088	0.875

Proportion of households changing to another social classes between two years, either proximate (number of change=1) or not (number of change>1). k is the parameter defined in Table 3.5.

Appendix IV

Table 4.1. Absolute Changes in Logarithmic Shadow Prices

	TP1		TP2		DP		TP1’		TP2’		DP’	
Period	*NV*	*V*	*NV*	*V*	*NV*	*V*	*NV*	*V*	*NV*	*V*	*NV*	*V*
87-88	0.193	0.154	0.286	0.315	0.188	0.258	0.135	0.108	0.108	0.141	0.130	0.186
88-89	0.214	0.173	0.119	0.351	0.208	0.287	0.148	0.122	0.118	0.157	0.142	0.210
89-90	0.282	0.217	0.131	0.441	0.274	0.358	0.197	0.152	0.157	0.196	0.188	0.262

Legend :
Average of the absolute values of the shadow prices changes: (1/CardH).Σ_h [(1/T-1) Σ_t Σ_i w_i.|$d\ln\pi_{iht}$|]
V = Violating Households
NV = Non-Violating Households
TP1 = Total population, budget share equalized : price marginal propensity = γ_{ii}
TP1’ = Total population, expenditure equalized: price marginal propensity = γ_{ii} - w_i
TP2 = Total population, budget share equalized and differenced income and squared income coefficients
TP2’ = Total population, expenditure equalized and differenced income and squared income coefficients
DP = Different populations (violating versus non-violating), budget share equalized
DP’ = Different populations (violating versus non-violating), expenditure equalized

Table 4.2. Indexes of changes in logarithmic shadow prices

	Index for all expenditures				Index for food at home		Index for clothing		Index for housing	
Violation (V) No Violation (NV)	**NV**	**NV**	**V**	**V**	**NV**	**V**	**NV**	**V**	**NV**	**V**
Indexes	I_q	I_a	I_q	I_a	$(d\ln\pi_{iht})^2$	$(d\ln\pi_{iht})^2$	$(d\ln\pi_{iht})^2$	$(d\ln\pi_{iht})^2$	$(d\ln\pi_{iht})^2$	$(d\ln\pi_{iht})^2$
1987-88	0.327	0.286	1.151	0.315	0.007	0.039	0.007	0.034	0.002	0.002
	(.006)		(.012)		(.0005)	(.0026)	(.0002)	(.0015)	(.0001)	(.0003)
1988-89	0.368	0.119	1.286	0.351	0.008	0.049	0.005	0.047	0.003	0.002
	(.004)		(.009)		(.0005)	(.0035)	(.0002)	(.0022)	(.0001)	(.0003)
1989-90	1.494	0.131	1.565	0.441	0.129	0.073	0.007	0.069	0.034	0.003
	(.003)		(.006)		(.015)	(.006)	(.0021)	(.0037)	(.0017)	(.0004)

Notes:
Estimation : Separately on pooled cross section and first differences. The variance of the difference (β_{cs} -β_{fd}) is taken as the sum of the variance of the two estimators. Estimation TP2.
Quadratic index : $I_q(\pi^v)$ = {(1/H).Σ_h [(1/T-1) Σ_t Σ_i w_i.$(d\ln\pi_{iht})^2$]}$^{0.5}$
Absolute index: $I_a(\pi^v)$ = (1/H).Σ_h [(1/T-1) Σ_t Σ_i w_i.|$d\ln\pi_{iht}$|]
Index variances : see Appendix VI.

APPENDIX V

5.1. REVELATION OF THE SHADOW PRICES

Two problems arise in the computation of shadow prices :

First, the shadow prices are computed in section 5.2 by equalizing the estimated budget share on cross-section plus the shadow price effect and the estimated budget share on time series. It assumes that the constant term is the same in the two estimations. Thus, the levels of the revealed shadow prices are disputable. However their variations between two periods, say $dlp_{vit} = \sum_i (d\pi_i/dZ_{1h}).dZ_{1ht}$ with $d\pi_i/dZ_{1h} = [\beta_i^{(c.s.)} - \beta_i^{(t.s.)}]/\gamma_{ii}$, are more reliable.

Second, it is also possible to reveal the variation of the shadow prices by equalizing the expenditures instead of the budget shares. Let us consider a logarithmic equation :

Log Ci = Z. $\beta_i^{(c.s.)} + u_i^{(c.s.)}$ = Z. $\beta_i^{(t.s.)} + E_i \log \pi_i + u_i^{(t.s.)}$ with E'_i the direct price elasticity and Ci the consumption of good i.

By separating the parameters of income from other determinants, we get $d\log\pi_i/dZ_{1h}$ = $[\beta_i^{(c.s.)} - \beta_i^{(t.s.)}]/(\gamma_{ii} - w_i)$ with w_i the average budget share for commodity i (this is easily proved by equalizing the income elasticities of this logarithmic specification with the elasticity computed by an AI specification).

Thus, to reveal the shadow prices, the two above hypotheses amount to divide the difference between cross-section and time series estimates either by γ_{ii} or by $(\gamma_{ii} - w_i)$.

5.2. ESTIMATION FOR THE TWO SUB-POPULATIONS

Suppose the estimates of the vector of parameters for some equation y = xβ + u, are $\hat{\beta}$ and $\hat{\beta}'$ for the estimations over two sub-populations of sizes n and n' with n < n'. The estimates can be written : $\hat{\beta} = \beta + t\sigma$ and $\hat{\beta}' = \beta' + t'\sigma$ with β and β' as the true parameters and σ the standard error of the parameters which is supposed to be the same for the two parameters. We suppose also that $\hat{\beta}$ and $\hat{\beta}'$ are independent, E(t) = E(t') = 0, $\sigma(t) = \sigma(t') = s$ and $\partial s/\partial N < 0$ where N is the population's size. This implies : $\hat{\beta} - \hat{\beta}' = (\beta - \beta') + (t-t')\sigma$ and $Var(\hat{\beta} - \hat{\beta}') = 2s^2\sigma^2$. The difference between the estimates on the two population is not biased with respect to 0, but decreases with the size of the sub-populations. Therefore, the difference between cross-sections and time-series estimates, which is a component of the shadow prices' formulas, is probably greater for the smaller population and that may artificially increase the variations of the shadow prices. Hence the shadow prices revealed by the estimations with respect to the violating households are over-estimated compared to those computed by a regression over the non-violating households (which are about ten times more numerous).

On the other hand, estimating the difference $(\beta^{(c.s.)} - \beta^{(t.s.)})$ on the whole population and estimating the shadow prices for the two populations with these estimates can be considered as misspecified. Indeed we proved that the consumption behavior differs between the two populations at least for the income parameters (Table 3.4). The income and squared income coefficients are well estimated for both populations, so that our preferred method to compute the shadow prices consists in estimating $\beta^{(c.s.)}$ and $\beta^{(t.s.)}$ on the whole population with dummies indicating the violating and non-violating for the income coefficients (row TP2 in Table 4.1, Appendix IV).

Appendix VI
Distribution of the Quadratic Index of Shadow Prices

1. *Computation of the variances :*

The quadratic index writes : $I_q(\pi^v) = \{(1/H).\Sigma_h [(1/T\text{-}1)\ \Sigma_t\ \Sigma_i\ w_i.(d\ln\pi_{iht})^2]\}^{0.5}$.

(i) The logarithmic virtual prices $d\ln\pi_{iht}$ are normally distributed as linear transforms of the differences between cross-sections and time-series parameters (which are, by assumption, normal). The variance of $d\ln\pi_{iht}$ is :
$Var(d\ln\pi_{iht}) = (dZ_{1ht})\ Var(\beta_B\text{-}\beta_W)(dZ_{1ht})'$,
with $Var(\beta_B\text{-}\beta_W) = Var(\beta_B) + Var(\beta_W)$: see Baltagi 1995, p.69.

(ii) The variance of $(d\ln\pi_{iht})^2$ can be computed using the formula, characterizing normal variables (here $d\ln\pi_{iht}$) :
(5.1) $Var((d\ln\pi_{iht})^2) = 2\sigma_i^4 + 4\ m_i^2\sigma_i^2$
with m_i and σ_i respectively the mean and standard-error of $d\ln\pi_i$. Thus, under an independence assumption, the variance of the square of the quadratic index I_q is the weighed average of these variances:
$Var[\Sigma_i w_i\ (d\ln\pi_{iht})^2] = \Sigma_i w_i^2\ [2\sigma_i^4 + 4\ m_i\sigma_i^2]$.

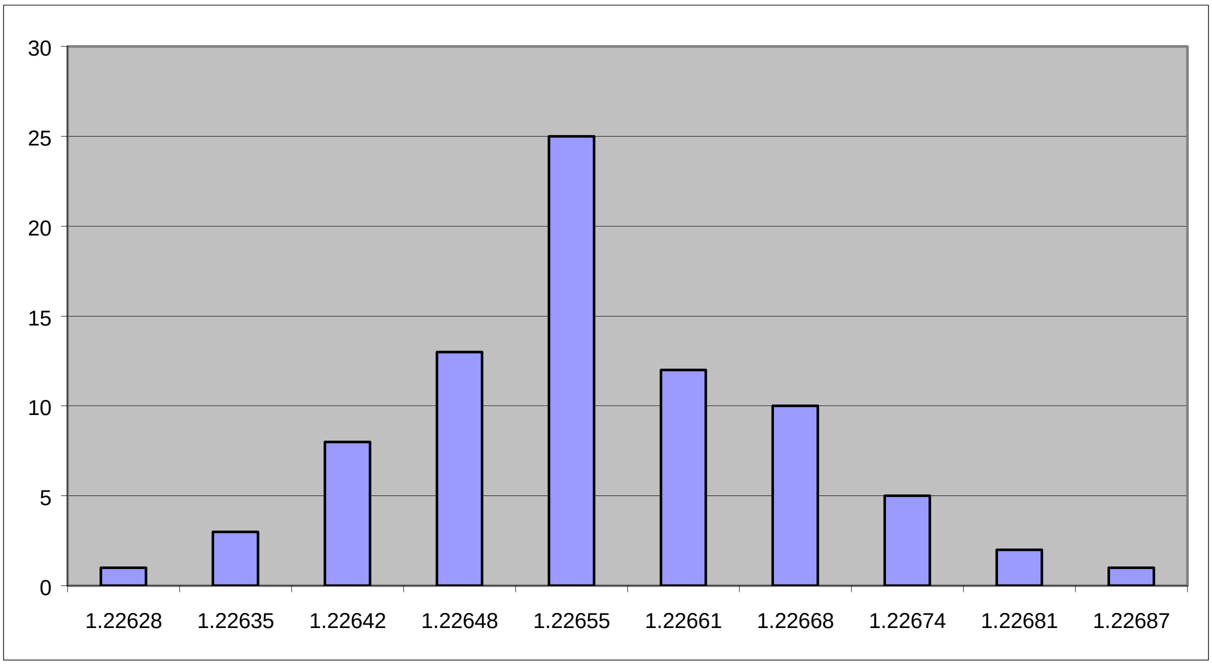
(iii) Finally, we assume (hypothesis 1.iii) that I_q is normal, so that the formula (5.1) applies also to this index and its square : $Var(I_q^2) = 2(VarI_q)^2 + 4(E(I_q))^2.Var(I_q)$. The positive solution for this equation writes $Var(I_q) = -\ I_q^2 + [I_q^4 + 0.5\ Var(I_q^2)]^{0.5}$. The different elements of this formula are taken as the average of I_q^2 and its variance over each sub-population composed respectively of the violating and non-violating households.

2. *Distribution of the quadratic index I_q :*

As a weighted average of non-centered heteroskedastic normal variables, I_q has no known distribution (let us point out however that there exist in mathematical-statistics - in very special cases unfortunately - some non-centered limit theorems). According to the tests ($\beta_1 = 7.42$ and $\beta_2 = 11.25$ are higher than 1.96 and the Jarque-Bera statistic s = 181.58 is higher than the limit of a $\chi^2(2)$), the distribution of I_q is not a normal distribution. Nevertheless we will assume *(see hypothesis 1.iii above)* I_q to be normal since a Monte Carlo experiment for 300 households and 80 draws shows that its histogram is close to the one of a normal distribution. Therefore we use the computed variances to test whether the quadratic index is significantly different from zero, and greater for the violating.

Hypothesis 1.iii is of course disputable.

Histogram of I_q



Legend.

X-axis : *Iq.*

Y-axis : *% of population.*